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Enhanced subleading structure functions in semileptonic B decay

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Abstract

The charged lepton spectrum in semileptonic $B \to X_u \ell \bar{\nu}$ decay near maximal lepton energy receives important corrections from subleading structure functions that are formally suppressed by powers of $\Lambda_{\rm QCD}/m_b$ but are enhanced by numerical factors. We investigate the series of higher order terms which smear over a region of width $\Delta E_\ell \sim \Lambda_{\rm QCD}$ near the endpoint the contributions proportional to $\delta(E_\ell - m_b/2)$ times (i) the matrix element of the chromomagnetic operator, and (ii) four-quark operators. These contribute to the total rate at the few percent level, but affect the endpoint region much more significantly. Implications for the determination of $|V_{ub}|$ are discussed.

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I. INTRODUCTION

Extracting the CKM matrix element $|V_{ub}|$ from the inclusive decay $B \to X_u \ell \bar{\nu}$ is complicated due to the large background from the decay $B \to X_c \ell \bar{\nu}$. A way to remove this background is to impose a lower cut on the charged lepton energy [1], restricting it to be within a few hundred MeV of the kinematic endpoint. Unfortunately, in this region of phase space there are large corrections to the decay rate due to the b quark distribution function in the B meson, which introduces model dependence into the extraction of $|V_{ub}|$. This uncertainty can be reduced by using the $B \to X_s \gamma$ photon spectrum as an input [2, 3], which was recently carried out by the CLEO collaboration [4].

The structure function which describes the lepton endpoint region in $B \to X_u \ell \bar{\nu}$ and the photon endpoint region in $B \to X_s \gamma$ arises due to an infinite series of formally higher order terms in the operator product expansion (OPE) becoming leading in the endpoint region. Corrections to this description in the endpoint region are only suppressed by one power of $\Lambda_{\rm QCD}/m_b$ (although nonperturbative effects in the total rate are suppressed by $\Lambda_{\rm QCD}^2/m_b^2$), but can be enhanced by other factors, as explained below. We concentrate on corrections which describe the smearing of certain terms in the OPE of order $(\Lambda_{\rm QCD}/m_b)^2\, \delta(1-y)$ and $(\Lambda_{\rm QCD}/m_b)^3\, \delta(1-y)$, where $y = 2E_\ell/m_b$. There are important terms at such orders, enhanced by numerical factors, which contribute to the $B \to X_u \ell \bar{\nu}$ rate at the several percent level. Since these contributions are concentrated near the endpoint, their effects on the rate in a restricted region can be an order of magnitude larger.

The $B \to X_u \ell \bar{\nu}$ lepton spectrum, including dimension-5 operators [5] and neglecting perturbative corrections, is given by

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}y} = \frac{G_F^2\, m_b^5\, |V_{ub}|^2}{192\,\pi^3} \bigg\{ 2y^2(3-2y)\,\theta(1-y) - \frac{2\lambda_2}{m_b^2} \Big[\frac{11}{2}\,\delta(1-y) - y^2(6+5y)\,\theta(1-y) \Big] \\ - \frac{2\lambda_1}{m_b^2} \Big[\frac{1}{6}\,\delta'(1-y) + \frac{1}{6}\,\delta(1-y) - \frac{5}{3}\,y^3\,\theta(1-y) \Big] + \ldots \bigg\}, \tag{1}$$

where

$$\lambda_1 = \frac{1}{2} \langle B(v)|\, \bar{b}_v\, (iD)^2\, b_v\, |B(v)\rangle\,, \qquad \lambda_2 = \frac{1}{6} \langle B(v)|\, \bar{b}_v\, \frac{g_s}{2}\, \sigma_{\mu\nu}\, G^{\mu\nu}\, b_v |B(v)\rangle\,, \tag{2}$$

and b_v denotes the b quark field in HQET. While λ_1 is not known precisely, the $B^* - B$ mass difference implies that $\lambda_2 = 0.12\,\mathrm{GeV}^2$. Of the terms in Eq. (1), the leading order (in $\Lambda_{\rm QCD}/m_b$) structure function contains $2[\theta(1-y) - \lambda_1/(6m_b^2)\,\delta'(1-y) + \ldots]$. The derivative of the same combination occurs in the $B \to X_s\gamma$ photon spectrum [6], given by

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}x} = \frac{G_F^2\, m_b^5\, |V_{tb}V_{ts}^*|^2\, \alpha\, C_7^2}{32\,\pi^4} \left[\left(1 + \frac{\lambda_1 - 9\lambda_2}{2m_b^2}\right) \delta(1-x) - \frac{\lambda_1 + 3\lambda_2}{2m_b^2}\, \delta'(1-x) - \frac{\lambda_1}{6m_b^2}\, \delta''(1-x) + \ldots \right], \tag{3}$$

where $x = 2E_\gamma/m_b$. The effect of the leading order structure function on the semileptonic decay is reviewed in Sec. II. At subleading order, proportional to $\delta(1-y)$ in Eq. (1) and to $\delta'(1-x)$ in Eq. (3), possibly the most striking difference is between the terms involving λ_2, whose coefficient is 11/2 in Eq. (1) and 3/2 in Eq. (3). Because of the 11/2 factor, the $\lambda_2\, \delta(1-y)$ term is important in the lepton endpoint region and it contributes about -5.5% to the total $B \to X_u\ell\bar\nu$ rate. This contribution is investigated in Sec. III.

At order $\Lambda_{\rm QCD}^3/m_b^3$, there are dimension-6 four-quark operators in the OPE,

$$O_{V-A} = (\bar b_v \gamma^\mu P_L u)\, (\bar u \gamma_\mu P_L b_v), \qquad O_{S-P} = (\bar b_v P_L u)\, (\bar u P_L b_v), \tag{4}$$

where $P_L = (1-\gamma_5)/2$. Compared to the lower dimensional operators discussed so far, these are enhanced by $16\pi^2$ and also enter the $B \to X_u\ell\bar\nu$ electron spectrum [7] as delta function contributions at the endpoint

$$\frac{\mathrm{d}\Gamma_{(6)}}{\mathrm{d}y} = -\frac{G_F^2\, m_b^2\, |V_{ub}|^2}{12\pi}\, f_B^2\, m_B\, (B_1 - B_2)\, \delta(1-y)\,. \tag{5}$$

Here f_B is the B meson decay constant and $B_{1,2}$ parameterize the matrix elements of the operators in Eq. (4) between B meson states,

$$\frac{1}{2} \langle B | O_{V-A} | B \rangle = \frac{f_B^2\, m_B}{8}\, B_1, \qquad \frac{1}{2} \langle B | O_{S-P} | B \rangle = \frac{f_B^2\, m_B}{8}\, B_2\,. \tag{6}$$

If one assumed factorization and used the vacuum saturation approximation then these contribution would vanish, since $B_1 = B_2 = 1$ (0) for charged (neutral) B mesons. There is no reason to believe that factorization holds exactly, and deviations from it at the 10% level are quite possible [7]. With such an estimate (i.e., $|B_1 - B_2| = 0.1$, and $f_B = 200\,\mathrm{MeV}$), the four-quark operators contribute of order 3% to the total $B \to X_u\ell\bar\nu$ rate. This contribution, which is absent from the $B \to X_s\gamma$ rate, is investigated in Sec. IV.

Section V contains our conclusions and the implications for the determination of $|V_{ub}|$.

II. THE LEADING ORDER STRUCTURE FUNCTION

Near the endpoint region of the $B \to X_u\ell\bar\nu$ lepton energy spectrum, the OPE is effectively an expansion in $\Lambda_{\rm QCD}/[m_b(1-y)]$ instead of $\Lambda_{\rm QCD}/m_b$. Thus, within $\Delta E_\ell \sim \Lambda_{\rm QCD}$ from the endpoint, there is an infinite series of terms of the form

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}y} = \frac{G_F^2 |V_{ub}|^2\, m_b^5}{192\,\pi^3} \sum_{n=0}^{\infty} \frac{A_n}{m_b^n\, n!}\, 2\,\theta^{(n)}(1-y)\,, \tag{7}$$

which are equally important when smeared over $\Delta y \sim \Lambda_{\rm QCD}/m_b$ [2]. Here A_n are given by the matrix elements

$$\frac{1}{2}\langle B(v)|\,\bar b_v\, iD_{\alpha_1}\ldots iD_{\alpha_n}\, b_v|B(v)\rangle = A_n\, v_{\alpha_1}\ldots v_{\alpha_n} + (\text{other tensor structures})\,, \tag{8}$$

and satisfy $A_0 = 1$, $A_1 = 0$, $A_2 = -\lambda_1/3$, etc. These terms can be formally resummed into a nonperturbative structure function for the b quark,

$$f(k_+) = \frac{1}{2}\langle B(v)|\,\bar b_v\, \delta(k_+ - iD\cdot n)\, b_v|B(v)\rangle\,, \tag{9}$$

where n is a light-like vector satisfying $n \cdot v = 1$ and $n^2 = 0$. The moments of $f(k_+)$ satisfy $\int \mathrm{d}k_+\, f(k_+)\, k_+^n = A_n$. This is convenient for model calculations because the effects of the distribution function can be included by replacing m_b by $m_b^* \equiv m_b + k_+$, and integrating over k_+,

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_\ell} = \int \mathrm{d}k_+\, f(k_+)\, \frac{\mathrm{d}\Gamma_{\rm p}}{\mathrm{d}E_\ell}\bigg|_{m_b\to m_b^*}\,, \tag{10}$$

where $\mathrm{d}\Gamma_{\rm p}/\mathrm{d}E_\ell$ is the parton-level spectrum. In the region we consider in this paper, $E_\ell > 2\,\mathrm{GeV}$, taking the parton level spectrum as $2y^2(3-2y)\,\theta(1-y)$ or just the leading twist part of it, $2\,\theta(1-y)$, makes a negligibly small difference numerically, and therefore we will use the latter. (Perturbative order α_s corrections are also neglected.)

For purposes of illustration, we will use a simple model for the structure function which has three parameters,

$$f(k_+) = \frac{1}{\bar\Lambda}\, \frac{a^{ab}}{\Gamma(ab)}\, (1-x)^{ab-1}\, e^{-a(1-x)}\, \theta(1-x)\,, \qquad x = \frac{k_+}{\bar\Lambda}\,. \tag{11}$$

The parameterization of Ref. [8] corresponds to $b \to 1$ and $a \to a+1$. The b parameter is introduced because the first moment of the subleading structure function that occurs in Sec. IV need not vanish, contrary to the leading order structure function in Eq. (9) and the subleading structure function that follows from the one-gluon matrix element discussed in Sec. III. The first three moments of $f(k_+)$ are given by

$$\begin{aligned} &\int \mathrm{d}k_+\, f(k_+) = 1\,, \qquad \int \mathrm{d}k_+\, f(k_+)\, k_+ = \bar\Lambda\,(1-b)\,, \\ &\int \mathrm{d}k_+\, f(k_+)\, k_+^2 = \bar\Lambda^2 \left[(1-b)^2 + \frac{b}{a}\right]. \end{aligned} \tag{12}$$

Lepton energy intervals	Fraction of events in the endpoint region			
	No smearing	$\lambda_1 = -0.1\,\text{GeV}^2$	$\lambda_1 = -0.3\,\text{GeV}^2$	$\lambda_1 = -0.5\,\text{GeV}^2$
$2.0\,\text{GeV} < E_\ell$	32.6%	32.6%	32.9%	33.4%
$2.1\,\text{GeV} < E_\ell$	24.3%	24.3%	24.8%	25.7%
$2.2\,\text{GeV} < E_\ell$	15.9%	16.1%	17.2%	18.3%
$2.3\,\text{GeV} < E_\ell$	7.5%	8.4%	10.2%	11.6%
$2.4\,\text{GeV} < E_\ell$	n/a	2.6%	4.6%	5.9%

TABLE I: Fractional rate in the endpoint region without and with resummation. For the structure function we used Eq. (11) with $b = 1$, $\bar\Lambda = 0.5\,\text{GeV}$, and λ_1 as shown in the Table.

For the leading order structure function we will use Eq. (11) with $b = 1$, $\bar\Lambda = 0.5\,\text{GeV}$, and $\lambda_1 = -3\bar\Lambda^2/a = -0.3\,\text{GeV}^2$, which are in the ballpark of the recent CLEO determinations [9].

The effect of resumming the leading singularities is illustrated in Table I. For the structure function we used Eq. (11) with $b = 1$, $\bar\Lambda = 0.5\,\text{GeV}$, and three different values of $\lambda_1 (= -3\bar\Lambda^2/a)$. This Table illustrates that the effect of the resummation becomes small as the lower limit of the lepton energy interval is lowered towards 2 GeV. Note that we have neglected perturbative corrections (this is probably the main reason why the numbers in the $\lambda_1 = -0.3\,\text{GeV}^2$ column differ from those in Ref. [4]). The value of the $\bar\Lambda$ parameter also affects the importance of resumming the singular terms. Of course the precise determination of $\bar\Lambda$ (i.e., the b quark mass) is crucial for any extraction of $|V_{ub}|$ from inclusive B decays.

III. SMEARING OF THE $\lambda_2\,\delta(1-y)$ CONTRIBUTIONS

In this section we investigate terms proportional to $\lambda_2\,\delta(1-y)$ in the $B \to X_u \ell\bar\nu$ lepton spectrum (or to $\lambda_2\,\delta'(1-x)$ in the $B \to X_s\gamma$ photon spectrum), which are subleading in the twist expansion, and therefore do not necessarily drop out from the relation between the two spectra. Such corrections originate from three sources (for details see [10]):

(i) corrections to the vanishing of the forward scattering matrix element of any dimension-4 operator, $\langle B(v)|\, \bar b_v\, iD^\alpha\, b_v\, |B(v)\rangle = 0$, due to higher order terms in the HQET Lagrangian;

(ii) corrections to the $b(x) = e^{-im_b v\cdot x}\, b_v(x)$ relation between the b quark field in QCD and in HQET;

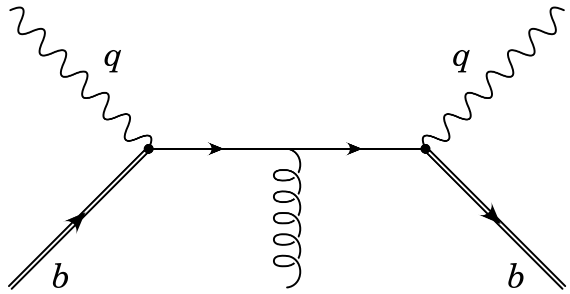

FIG. 1: The one-gluon matrix element.

(iii) corrections to the leading contribution of the one-gluon matrix element in Fig. 1.

Correspondingly, there are three series of higher dimensional terms in the OPE that can be resummed into three subleading structure functions that describe the smearing of these terms in regions of widths $\Delta E_\ell \sim \Lambda_{\rm QCD}$ near the endpoint. These subleading structure functions that smear the (i), (ii), and (iii) terms discussed above are denoted in Ref. [11] by $t(\omega)$, $h_1(\omega)$, and $h_2(\omega_1, \omega_2)$, respectively. Since none of them are known, we will focus on possible effects of only one of them, which has the largest coefficient.

Of these subleading structure functions, the one related to (i) is universal and may be absorbed into the redefinition of the leading order structure function. Therefore, this function drops out from the relation between the $B \to X_u \ell \bar\nu$ lepton spectrum and the $B \to X_s \gamma$ photon spectrum. To gain insight into the relative importance of the other two structure functions, note that the coefficient of the $(\lambda_2/m_b^2)\, \delta'(1-x)$ term in Eq. (3) arises from the (i)–(iii) terms respectively as $3/2 = (3-2+2)/2$ [11], whereas the corresponding decomposition of the coefficient of the $(\lambda_2/m_b^2)\, \delta(1-y)$ term in Eq. (1) is $11/2 = (3+2+6)/2$. Therefore, we will focus on the subleading structure function that smears the one-gluon matrix element (iii).

The one-gluon matrix element in Fig. 1 is given by the imaginary part of the B meson matrix element of

$$\bar b\, \gamma_\mu \left[\frac{(m_b \not{v} + \not{k} - \not{q})\, (i g_s T^A \varepsilon^{A\lambda *} \gamma_\lambda)\, (m_b \not{v} + \not{k}' - \not{q})}{(m_b v + k - q)^2\, (m_b v + k' - q)^2} \right] \gamma_\nu P_L b\,, \tag{13}$$

where ε is the gluon polarization vector. The gluon field strength arises from the part antisymmetric in the gluon momentum ($p = k - k'$) and polarization, $p^\mu T^A \varepsilon^{A\nu *} \to -(i/2) G^{\mu\nu}$. The series of most important terms in the OPE in the endpoint region is obtained by expanding in k and k', and keeping the part where in each additional order in $\Lambda_{\rm QCD}/m_b$ the

Lepton energy intervals	Fractional correction, $\delta\Gamma/\Gamma$		
	$a = 2.5$	$a = 1$	$a = 10$
$2.0\,\mathrm{GeV} < E_\ell$	-9%	-9%	-10%
$2.1\,\mathrm{GeV} < E_\ell$	-12%	-11%	-13%
$2.2\,\mathrm{GeV} < E_\ell$	-16%	-15%	-18%
$2.3\,\mathrm{GeV} < E_\ell$	-24%	-23%	-27%
$2.4\,\mathrm{GeV} < E_\ell$	-38%	-42%	-33%

TABLE II: Fractional corrections to the rate in the endpoint region due to the $\lambda_2\,\delta(1-y)$ term from the one-gluon matrix element. The subleading structure function is modelled by Eq. (11) with $b = 1$, $\bar\Lambda = 0.5\,\mathrm{GeV}$, and the different values of a shown in the Table. For the leading structure function $a = 2.5$ is held fixed (corresponding to $\lambda_1 = -0.3\,\mathrm{GeV}^2$), and $b = 1$ and $\bar\Lambda = 0.5\,\mathrm{GeV}$.

contribution to the rate becomes more singular [2, 12]. These terms enter proportional to matrix elements where each additional covariant derivative gives v^α tensor structure. In analogy with λ_2 in Eq. (2), we define

$$\begin{aligned}\sum_{m=0}^{n} \frac{1}{2}\langle B(v)|\, \bar b_v\, iD_{\alpha_1}\ldots iD_{\alpha_m}\Big[\frac{g_s}{2}\,\sigma_{\mu\nu}\,G^{\mu\nu}\Big] iD_{\alpha_{m+1}}\ldots iD_{\alpha_n}\, b_v|B(v)\rangle \\ = 3(n+1)\,\lambda_2\, X_n\, v_{\alpha_1}\ldots v_{\alpha_n} + (\text{other tensor structures})\,,\end{aligned} \qquad (14)$$

The normalization is chosen such that $X_0 = 1$, and $X_1 = 0$ because of the heavy quark equation of motion. The higher order matrix elements are unknown. The resulting contribution to the $B \to X_u \ell \bar\nu$ lepton spectrum is

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}y} = -\frac{G_F^2 |V_{ub}|^2\, m_b^3}{192\,\pi^3}\, 6\lambda_2 \sum_{n=0}^{\infty} \frac{X_n}{m_b^n\, n!}\, \delta^{(n)}(1-y)\,. \qquad (15)$$

As for the leading order structure function contribution, the effects of Eq. (15) can be modelled by smearing the $n = 0$ term with a function $f(k_+)$, as in Eq. (10), where the moments of $f(k_+)$ must satisfy $\int \mathrm{d}k_+\, f(k_+)\, k_+^n = X_n$.

In Table II we give numerical estimates of the effect of these corrections on the $B \to X_u \ell \bar\nu$ rate in the endpoint region. We use for the leading order structure function the model in Eq. (11) with $b = 1$, $\bar\Lambda = 0.5\,\mathrm{GeV}$ and $\lambda_1 = -0.3\,\mathrm{GeV}^2$. Since the leading structure function smears $\theta(1-y)$, the sensitivity to the precise values of its parameters is relatively smaller than in the case of the subleading structure functions that smear contributions proportional

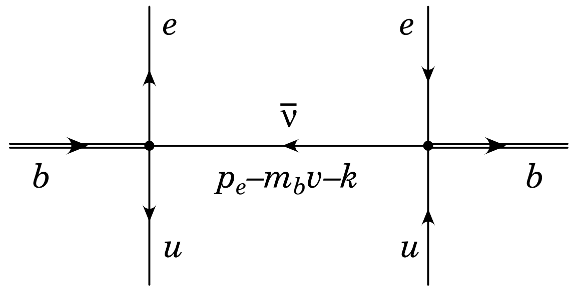

FIG. 2: Time ordered product that yields four-quark operators and higher dimension terms.

to $\delta(1-y)$. For the subleading structure function we use Eq. (11) with $b = 1$, $\bar{\Lambda} = 0.5\,\mathrm{GeV}$ and with different values of the a parameter. Using the $B \to X_s\gamma$ photon spectrum as an input only cancels a third of these corrections, and furthermore, there is another subleading structure function that contributes comparably and with the same sign as the one shown in Table II. The sign of these corrections is negative in all entries in Table II, because the subleading structure function is assumed to be positive for all values of k_+ and because the contribution of this correction to the total rate is negative.

IV. SMEARING OF THE FOUR-QUARK OPERATORS' CONTRIBUTION

There are also important contribution to the lepton endpoint region from the four-quark operators in Eq. (4). Compared to the b quark decay rate, these contributions in the lepton endpoint region are suppressed by $\Lambda_{\mathrm{QCD}}^2/m_b^2$ but are enhanced by $16\pi^2$ (their suppression is $\Lambda_{\mathrm{QCD}}^3/m_b^3$ in the total rate). There is again an infinite series of higher dimensional terms in the OPE which are equally important in the region of lepton energies within order Λ_{QCD} from the endpoint. These are obtained from the amplitude in Fig. 2, which is proportional to

$$(\bar{b}\gamma_\mu P_L u)\left[\frac{\bar{\ell}\gamma^\nu P_L\, i(m_b \not{v} + \not{k} - \not{p}_\ell)\, \gamma^\mu P_L \ell}{(m_b v + k - p_\ell)^2}\right](\bar{u}\gamma_\nu P_L b) \tag{16}$$

The series of most singular terms which are equally important in the endpoint region of the lepton spectrum is obtained by expanding in k and taking the imaginary part,

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}y} \propto \sum_{n=0}^{\infty} \delta^{(n)}(1-y)\, \frac{2^n}{n!\, m_b^n}\, (g^{\mu\nu} - v^\mu v^\nu)\, (v-\hat{p}_\ell)^{\alpha_1} \ldots (v-\hat{p}_\ell)^{\alpha_n}$$
$$\times \langle B(v)|(\bar{b}_v\gamma_\mu P_L u)\, iD_{\alpha_1} \ldots iD_{\alpha_n} (\bar{u}\gamma_\nu P_L b_v)|B(v)\rangle\,. \tag{17}$$

Here $\hat{p}_\ell = p_\ell/m_b$, and the $(g^{\mu\nu} - v^\mu v^\nu)$ term arises because in the kinematic region we are

Lepton energy intervals	Fractional correction, $|\delta\Gamma|/\Gamma$		
	$b=1/2$	$b=1$	$b=3/2$
$2.0\,\mathrm{GeV} < E_\ell$	9%	9%	8%
$2.1\,\mathrm{GeV} < E_\ell$	12%	12%	10%
$2.2\,\mathrm{GeV} < E_\ell$	17%	16%	12%
$2.3\,\mathrm{GeV} < E_\ell$	28%	23%	15%
$2.4\,\mathrm{GeV} < E_\ell$	57%	37%	18%

TABLE III: Fractional corrections to the rate in the endpoint region due to the smeared four-quark operators. The first moment of this subleading structure function need not vanish, so it is modelled by Eq. (11) with $\bar\Lambda = 0.5\,\mathrm{GeV}$, $a = 2.5$ and the different values of b shown in the Table. For the leading structure function we used $b = 1$, $\bar\Lambda = 0.5\,\mathrm{GeV}$, $a = 2.5$ (corresponding to $\lambda_1 = -0.3\,\mathrm{GeV}^2$).

considering v^μ dotted into the lepton tensor gives a subleading contribution. Since $v - \hat p_\ell$ defines a light-cone direction, $(v-\hat p_\ell)^2$ yields a suppression by at least one power of $\Lambda_{\mathrm{QCD}}/m_b$. Therefore, we may write

$$\begin{aligned}&\frac{1}{2}\langle B(v)|(\bar b_v\gamma_\mu P_L u)\, iD_{\alpha_1}\dots iD_{\alpha_n}(\bar u\gamma_\nu P_L b_v)|B(v)\rangle\,(g^{\mu\nu}-v^\mu v^\nu)\\&\qquad = Y_n\, v_{\alpha_1}\dots v_{\alpha_n} + (\text{other tensor structures})\,,\end{aligned}\tag{18}$$

where the other tensor structures yield subleading contributions. Collecting all factors, the contribution to the lepton spectrum is

$$\begin{aligned}\frac{\mathrm{d}\Gamma}{\mathrm{d}y} &= -\frac{G_F^2|V_{ub}|^2\, m_b^2}{3\,\pi}\,(g^{\mu\nu}-v^\mu v^\nu)\,\langle B(v)|(\bar b_v\gamma_\mu P_L u)\,\delta(1-y-iD\cdot n)\,(\bar u\gamma_\nu P_L b_v)|B(v)\rangle\\ &= -\frac{2G_F^2|V_{ub}|^2\, m_b^2}{3\pi}\sum_{n=0}^{\infty}\frac{Y_n}{m_b^n\, n!}\,\delta^{(n)}(1-y)\,.\end{aligned}\tag{19}$$

The first term in the expansion of the delta function reproduces Voloshin's result in Eq. (5), since $Y_0 = (\langle B|O_{V-A}|B\rangle - \langle B|O_{S-P}|B\rangle)/2 = (B_1 - B_2) f_B^2\, m_B/8$.

An important difference between the structure function for the four-quark operators and those discussed explicitly in Secs. II and III is that in the present case the first moment of $f(k_+)$ need not vanish. The importance of this difference is illustrated in Table III. To obtain these estimates we held the leading order structure function fixed, as discussed previously. For the subleading structure function we use Eq. (11) with $\bar\Lambda = 0.5\,\mathrm{GeV}$ and $a = 2.5$ held fixed, while varying the b parameter which determines the first moment. Since such contributions do not occur in radiative B decay, using the $B \to X_s\gamma$ photon spectrum as an input does not reduce these corrections. The sign of this correction is undetermined, and the magnitude is directly proportional to the assumed violation of factorization (we used $|B_1 - B_2| = 0.1$ and $f_B = 200\,\mathrm{MeV}$). It may be possible to address the size of factorization violation using lattice QCD [13].

V. CONCLUSIONS

We investigated subleading structure function contributions to $B \to X_u\ell\bar{\nu}$ that are important in the region $E_\ell > 2\,\mathrm{GeV}$, and could each alter significantly the relation between the lepton spectrum and the $B \to X_s\gamma$ photon spectrum used for the extraction of $|V_{ub}|$. We concentrated on two subleading structure functions.

The first series of corrections, considered in Sec. III, is related to the smearing of the part of the $(\lambda_2/m_b^2)\,\delta(1-y)$ contributions to the $B \to X_u\ell\bar{\nu}$ lepton spectrum originating from the one-gluon matrix element in Fig. 1. This is formally an order $\Lambda_{\mathrm{QCD}}/m_b$ correction in the endpoint region, however it enters with an enhanced coefficient, 6. Its contribution to the total $B \to X_u\ell\bar{\nu}$ rate is known to be -3%, and estimates of how it affects the lepton spectrum integrated over different intervals near the endpoint are shown in Table II.

The second series of corrections, considered in Sec. IV, is related to the smearing of four-quark operator contributions. This is formally an order $\Lambda_{\mathrm{QCD}}^2/m_b^2$ correction in the endpoint region, however it is enhanced by $16\pi^2$. Contrary to the previous case, the sign of this contribution is unknown, and its magnitude is also uncertain. It could be a $\sim 3\%$ contribution to the total $B \to X_u\ell\bar{\nu}$ rate, and estimates of how it affects the lepton spectrum integrated over different intervals near the endpoint are shown in Table III.

These corrections could each affect the value of $|V_{ub}|$ extracted from the lepton endpoint analysis above the 10% level, even when it is combined with the measurement of the $B \to X_s\gamma$ photon spectrum. While varying the lower end of the energy interval between $2\,\mathrm{GeV}$ and $2.4\,\mathrm{GeV}$, as done in the recent CLEO analysis, provides some indication that these corrections are probably not much larger than our estimates, the uncertainty is reliably reduced only if the lower limit of the lepton energy interval is near $2\,\mathrm{GeV}$, in which case the lepton energy cut no longer eliminates the charm background. However, if the $B \to X_u\ell\bar{\nu}$ lepton spectrum can be measured accurately down to $2\,\mathrm{GeV}$, then it is likely that a resummation of singular terms in the OPE in the endpoint region is not necessary. In that case, knowledge of the $B \to X_s\gamma$ photon spectrum is not needed[1] and $|V_{ub}|$ can be extracted without a resummation of the leading singular terms. In the $E_\ell \gtrsim 2.2\,\mathrm{GeV}$ region, where the resummation of the leading singularities in the OPE is important, so is the uncertainty related to the subleading structure functions discussed in this paper. With more precise $B \to X_u\ell\bar{\nu}$ and $B \to X_s\gamma$ data in the region above $2.2\,\mathrm{GeV}$, it may be possible to observe these corrections.

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